

Study on oscillating flow of moderate kinetic Reynolds numbers using complex velocity model and phase Doppler anemometer



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HIGHLIGHTS

- PDA experiments were conducted to study oscillating flows with Re_ω up to 1800.
- SiO_2 of 0.9 μm diameter were suggested as tracer particles.
- A complex method was proposed to gain velocity distributions of oscillating flows.
- The average deviation between analytical and experimental results was $\sim 10\%$.

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ABSTRACT

Oscillating flows have fundamental effects on the heat and mass transfer of Stirling engines and gas springs. A complex velocity model was proposed to describe the fully-developed laminar oscillating flow through introducing complex functions and simplified control equations. The maximum dimensionless velocity decreased and the smooth core area of velocity expanded as the kinetic Reynolds number increased, which led the Stokes layer became thinner. Experiments were carried out in an apparatus consisting of piston-cylinder space connected to a straight duct. Instantaneous velocities of oscillating flows for Re_ω value of 230, 900 and 1800 were measured by a phase Doppler anemometer (PDA), where the effects of responding time, accuracy and features of tracer particles were analyzed. The experiments indicated that the complex velocity model with Re_ω was suitable to describe the fully-developed laminar oscillating flow, and the average deviation was about 10%. For a moderate-value Re_ω , the ratios of maximum dimensionless velocity for $r = 0.85 R$ to $r = 0 R$ were 1.06–1.10, which were evidences of “annular effect”.

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1. Introduction

Oscillating flows, whose flow direction changes periodically, are applied in many energy fields. Stirling engines are typically kind of external combustion engines, which can be widely used in the fields of solar energy, biomass, and nuclear energy to produce mechanical power. Oscillating flow is the main property of Stirling engines, which has great influence on heat transfer and flow resistance [1]. As a core component of energy devices, e.g. refrigerator and internal combustion engines, gas springs are widely used to reduce resistance and vibration, and the gas-wall heat transfer and the hysteresis loss of gas springs are strong affected by oscillating flows [2]. Additionally, similar flows always appear in ocean wave energy, oscillating heat-pipes and solar collectors, etc. [3–5].

There is a big difference between oscillating flow and unidirectional flow. If the oscillating frequency is high enough, the maximum velocity occurs near wall instead of in the center, i.e. “annular effect”. This characteristic is much different from other flows [6].

The Stokes layer of oscillating flow is much thinner than that of steady flow because of its unsteady condition [7], leading to high heat transfer efficiency. It was noticed that the core section of the velocity profile was smooth at a large Re_ω number. As Re_ω increased, this smooth area expanded, indicating that the Stokes layer becomes thinner [8]. Kurzweg and Zhao [9] found the effective thermal diffusivity of oscillating flows was up to 17,900 times than that of unidirectional flows. Mathematical models have been proposed to analyze and resolve oscillatory velocity fields [6]. Uchida [10] reported an analytical study to a pulsating laminar flow of incompressible fluid and obtained results of low-value Re_ω numbers (4, 36, 100 and 400). He found that the magnitude of velocity of oscillatory flow for low values of Re_ω varied periodically

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Nomenclature

a	the acceleration (g)	R	radius of pipe (m)
A	dimensionless oscillation amplitude (/)	T	temperature (K)
c	specific heat of fluid (kJ/(kg K))	$x_{\max}$	maximum oscillation amplitude (m)
D	hydraulic diameter of test duct (m)	t	time (s)
D_o	outer diameter of test duct (m)	u	velocity of temporal flow (m)
f	frequency of oscillation (Hz)	$u_{\max}$	maximum cross-section mean velocity of temporal flow (m)
i	imaginary number ($\sqrt{-1}$)	$\frac{U}{\vec{V}}$	dimensionless velocity (/)
J	bessel function	$\vec{V}$	velocity vector
k	heat transfer coefficient	Stk	the Stokes number (/)
p	pressure (p)		
Pr	the Prandtl number(/)		
$\Re$	real number		
$Re_{\max}$	maximum Reynolds number (/)		
Re_{ω}	kinetic Reynolds number (/)		
r	radial distance of pipe (m)		
θ	angle ($^{\circ}$)		
φ	shaft degree ($\varphi = \omega t$)		
ρ	fluid density (kg/m ³)		
ω	angular frequency (rad/s)		
μ	dynamic viscosity (pa s)		
ν	kinematic viscosity (m ² /s)		
Φ	fluid viscous dissipation		
τ	dimensionless time		
τ_p	the characteristic time of the particle		
ψ	dimensionless temperature		

Greek symbols

α	thermal diffusivity of fluid
α'	viscous length (/)
β	critical dimensionless parameter

Subscripts

0	zero order Bessel function
1	one order Bessel function
ma	maximum value
X	X-axis
y	Y-axis
z	Z-axis
$\sim, ^{\wedge}$	complex conjugates

and its phase was same to that of the pressure gradient. While for a flow of large Re_{ω} at the center of pipe, there was phase lag of 90° to the wave of pressure gradient and the amplitude diminished as frequency increased. Juárez [11] carried out a numerical simulation of transition to turbulence for a purely oscillatory channel flow using finite element space and reported pretty results. Zhao and Cheng [12] believed the characteristics of oscillating flow, including heat transfer coefficients were mainly affected by Re_{ω} , oscillatory amplitude of fluid, ratios of length to diameter and Prandtl numbers.

Considering that practical oscillating flows in heat exchanger with various geometry shapes were very complex, people always simplified them into flows in straight ducts with rigid walls for experimental study [6,13]. Hot wire anemometer (HWA), photo-electric detector, laser Doppler anemometer (LDA) and Particle Image Velocimetry (PIV) [14] are often used to measure oscillating-flow field. No tracer particles are required offers a transient resolution of acquired signals. Mao et al. [15] used a HWA to study velocity fluctuations related to vortex shedding phenomena behind thermo-acoustic stacks. Zhao and Cheng [16] investigated a reversing flow of air in a pipe using a HWA and obtained correlation equations of friction coefficient. The main errors of HWA were caused by probes which interference with the flow field. A LDA or PIV did not disturb flow field, and was easy to be used to measure a set of positions in the flow through tracer particles and optical methods. While the size distribution of tracer seeds could not obtain with these two methods.

The oscillating flows of heaters or regenerators of Stirling engines make the heat transfer coefficient reach $\sim 1 \times 10^4 \text{ W}/(\text{m}^2 \text{ K})$ [17], which is much greater than those of other heat exchangers. Some practical applications of oscillating flows in the field of heat transfer enhancement are listed in Table 1.

The dimensionless conservation of energy equation is given [13]:

$$\frac{\partial \psi}{\partial \tau} + \frac{A}{2} (\vec{V} \cdot \nabla) \psi = \frac{1}{Re_{\omega} Pr} (\nabla^2 \psi) \quad (1)$$

where, Pr is the Prandtl number with α being the thermal diffusivity of fluid. Thermal boundary layer thickness δ_T is expressed as,

$$\delta_T \sim \sqrt{\frac{\nu}{\omega}} \quad (2)$$

$$Nu \sim \sqrt{\frac{\omega}{\nu}} \quad (3)$$

From Eq. (1), it is found that the kinetic Reynolds number Re_{ω} in an oscillating heat transfer is important as well as Reynolds number for unidirectional steady flows. It can infer that the heat transfer coefficient k of an oscillating pipe flow would increase with Re_{ω} increasing. The thermal boundary layer thickness and the location of boundary layer separation become unstable, for flow velocity and direction change rapidly, which make the flow in turbulent region even with a low Reynolds number.

In this paper, a complex velocity model of fully developed oscillating flow was developed and relating analytical resolutions were obtained. Axial velocities and “annular effect” were tested experimentally by a phase Doppler anemometer (PDA) for oscillating flows of moderate-value Re_{ω} numbers (230, 900 and 1800) to validate the velocity model, in which kinetic Reynolds numbers were higher than other reports.

2. Complex velocity model and experimental test rig

2.1. Fundamental theories

An oscillating flow is unsteady and compressible in a pipe of circular section. It is supposed that an oscillating flow field is at an ambient temperature and a positive pressure, and then density, pressure, speed and viscosity of working fluid are unrelated to the temperature [18]. The control equations of cylindrical coordinates are expressed by [19], continuity equation is written by,

Table 1

Heat transfer enhancement in oscillating flows of applications.

Author	Application	Positive effects on heat transfer
Lambert et al. [5]	Solar collectors	The maxima effective thermal diffusivity reached $1.8 \times 10^6 \text{ W/cm}^2$, two orders of magnitude larger than those of liquid metal heat pipes at the condition of $\text{Re}_\omega = 5.656$, $\text{Pr} = 10$, $\omega = 300 \text{ rad/s}$ and a temperature gradient of 10 K/cm
Yuan [17]	Stirling engine regenerator	The heat transfer coefficient and the Nusselt number of oscillating flow of the Stirling engine regenerator were at the order of $1 \times 10^4 \text{ W/(m}^2 \text{ K)}$ and 10, respectively, for high pressure and high frequency conditions

$$\frac{\partial \rho}{\partial t} + \frac{1}{r} \left[\frac{\partial(r\rho V_r)}{\partial r} + \frac{\partial(\rho V_\theta)}{\partial \theta} + \frac{\partial(r\rho V_x)}{\partial x} \right] = 0 \quad (4)$$

monumental equation is written by,

$$\rho \left(\frac{DV_r}{Dt} - \frac{V_\theta^2}{r} \right) = \rho f_r + \frac{1}{r} \left[\frac{\partial(rp_{rr})}{\partial r} + \frac{\partial(p_{\theta r})}{\partial \theta} + \frac{\partial(rp_{xr})}{\partial x} - p_{\theta\theta} \right] \quad (5)$$

$$\rho \left(\frac{DV_\theta}{Dt} + \frac{V_r V_\theta}{r} \right) = \rho f_\theta + \frac{1}{r} \left[\frac{\partial(rp_{r\theta})}{\partial r} + \frac{\partial(p_{\theta\theta})}{\partial \theta} + \frac{\partial(rp_{x\theta})}{\partial x} + p_{r\theta} \right] \quad (6)$$

$$\rho \frac{DV_x}{Dt} = \rho f_x + \frac{1}{r} \left[\frac{\partial(rp_{rx})}{\partial r} + \frac{\partial(p_{\theta x})}{\partial \theta} + \frac{\partial(rp_{xx})}{\partial x} \right] \quad (7)$$

The ideal gas equation of state is written by,

$$p = \rho RT \quad (8)$$

where a stress tensor and a rate of strain tensor are given by,

$$p_{rr} = -p + 2\mu \left(\varepsilon_{rr} - \frac{1}{3} \nabla \cdot \vec{V} \right), \quad \varepsilon_{rr} = \frac{\partial V_r}{\partial r} \quad (9)$$

$$p_{\theta\theta} = -p + 2\mu \left(\varepsilon_{\theta\theta} - \frac{1}{3} \nabla \cdot \vec{V} \right), \quad \varepsilon_{\theta\theta} = \frac{1}{r} \frac{\partial V_\theta}{\partial \theta} + \frac{V_\theta}{r} \quad (10)$$

$$p_{xx} = -p + 2\mu \left(\varepsilon_{xx} - \frac{1}{3} \nabla \cdot \vec{V} \right), \quad \varepsilon_{xx} = \frac{\partial V_x}{\partial x} \quad (11)$$

$$p_{r\theta} = p_{\theta r} = 2\mu \varepsilon_{r\theta}, \quad \varepsilon_{r\theta} = \varepsilon_{\theta r} = \frac{1}{2} \left(\frac{\partial V_\theta}{\partial r} - \frac{V_\theta}{r} + \frac{1}{r} \frac{\partial V_r}{\partial \theta} \right) \quad (12)$$

$$p_{\theta x} = p_{x\theta} = 2\mu \varepsilon_{\theta x}, \quad \varepsilon_{\theta x} = \varepsilon_{x\theta} = \frac{1}{2} \left(\frac{\partial V_\theta}{\partial x} + \frac{1}{r} \frac{\partial V_x}{\partial \theta} \right) \quad (13)$$

$$p_{xr} = p_{rx} = 2\mu \varepsilon_{rx}, \quad \varepsilon_{rx} = \varepsilon_{xr} = \frac{1}{2} \left(\frac{\partial V_r}{\partial x} + \frac{1}{r} \frac{\partial V_x}{\partial r} \right) \quad (14)$$

The total derivative is expressed by,

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + V_r \frac{\partial}{\partial r} + \frac{V_\theta}{r} \frac{\partial}{\partial \theta} + V_x \frac{\partial}{\partial x} \quad (15)$$

The divergence of velocity is written by

$$\text{div } \vec{V} = \nabla \cdot \vec{V} = \frac{1}{r} \left[\frac{\partial(rV_r)}{\partial r} + \frac{\partial V_\theta}{\partial \theta} + \frac{\partial(rV_x)}{\partial x} \right] \quad (16)$$

Suppose tubes of heat exchangers were long enough, the investigated part would be a fully developed region, and the axial velocity distribution would be stable [8,9]. According to the research of Stirling engine heat exchangers, effects of compressibility of gas could be negligible [6,20]. Therefore, continuity equation is written by,

$$\frac{\partial V_x}{\partial x} = 0 \quad (17)$$

monumental equation is written by,

$$\frac{1}{v} \frac{\partial V_x}{\partial t} = \frac{\partial^2 V_x}{\partial r^2} + \frac{1}{r} \frac{\partial V_x}{\partial r} - \Delta p \quad (18)$$

where

$$\Delta p = \frac{1}{\mu} \frac{\partial p}{\partial z} \quad (19)$$

The velocity of oscillating flow in heat exchanger tubes is a function of time and location, given by,

$$V = V(t, r) \quad (20)$$

which is rewritten by,

$$V = V(t, r) = f(t) \cdot g(r) \quad (21)$$

As the oscillating flow is driven by sinusoidal motions of a piston $A \sin(\omega t)/2$, it is convenient to adopt the complex nation, and Eq. (21) can be written by,

$$V_x = \frac{A\omega}{2} \cos(\omega t) = \hat{V}_x(r) e^{i\omega t} + \tilde{V}_x(r) e^{-i\omega t} \quad (22)$$

The oscillating flow of fluid is driven by a piston instead of an imposed pressure drop [2,21]. It is supposed the oscillatory velocity and the pressure have the same phase lag according to the mechanism for oscillating flows [22].

$$\Delta p = \hat{p} e^{i\omega t} + \tilde{p} e^{-i\omega t} \quad (23)$$

Substituting (22) and (23) into (18),

$$\frac{i\omega}{v} \cdot \hat{V}_x(r) = \frac{\partial^2 \hat{V}_x(r)}{\partial r^2} + \frac{1}{r} \cdot \frac{\partial \hat{V}_x(r)}{\partial r} - \hat{p} \quad (24)$$

then,

$$\hat{V}_x(r) = c_1 J_0(\alpha' r) + c_2 Y_0(\alpha' r) + \frac{\hat{p}}{\alpha'^2} \quad (25)$$

where

$$\alpha' = \sqrt{-\frac{i\omega}{v}} \quad (26)$$

The boundary conditions of heat exchanger tubes are

$$V(0, t) = \text{finite velocity} \quad \text{at } r = 0$$

$$V(R, t) = 0 \quad \text{at } r = R$$

Therefore,

$$\hat{V}_x(r) = \frac{\hat{p}}{\alpha'^2} \left[1 - \frac{J_0(\alpha' r)}{J_0(\alpha' R)} \right] \quad (27)$$

The integral of velocity over cross sectional area is equal to the volumetric flow rate, which is written by,

$$\int_0^R 2\pi V r dr = q_v = \pi R^2 \frac{A\omega}{4} \quad (28)$$

The pressure term can be solved in terms of amplitude to yield as,

$$\hat{p} = \frac{A\omega\alpha'^2}{4 - \frac{8J_1(\alpha'R)}{R\alpha'J_0(\alpha'R)}} \quad (29)$$

Substituting (29) into (27) and (22), the velocity field is written by,

$$V_x(t, r) = A\omega \left\{ \left[\frac{1 - \frac{J_0(\alpha' r)}{J_0(\alpha' R)}}{4 - \frac{8J_1(\alpha' R)}{\alpha' R J_0(\alpha' R)}} \right] e^{i\omega t} + \left[\frac{1 - \frac{J_0(\alpha' r)}{J_0(\alpha' R)}}{4 - \frac{8J_1(\alpha' R)}{\alpha' R J_0(\alpha' R)}} \right] e^{-i\omega t} \right\} \quad (30)$$

then, theoretical value is given by,

$$V_x = \Re(V_x(t, r)) = A\omega \left[\frac{1 - \frac{J_0(\alpha' r)}{J_0(\alpha' R)}}{2 - \frac{4J_1(\alpha' R)}{\alpha' R J_0(\alpha' R)}} \right] \cos(\omega t) \quad (31)$$

To quantitatively evaluate the similarity between analytical resolutions and experimental results, a deviation was proposed, which is define by,

$$\text{deviation} = \frac{\text{experimental result} - \text{analytical resolution}}{\text{analytical resolution}} \quad (32)$$

2.2. PDA experimental rig

Fig. 1 shows a schematic diagram of experimental device. An oscillating flow is induced by a piston with sinusoidal motion, which is remodeled from a gasoline engine. The diameter and stroke of cylinder are 74 mm and 70 mm, respectively. The piston is driven by a 1.3 kW asynchronous motor with an adjustable speed range of 0.3–4 Hz. Working fluid (air) moves reversibly and periodically in the test section, which is made of two Plexiglas pipes, (a) $D_o = 30$ mm in outer diameter and 100 mm in length, (b) $D_o = 50$ mm in outer diameter and 200 mm in length. One end of the pipe is connected to cylinder of the reciprocating device, the other end is closed.

Velocity field of oscillating flows in the pipe is measured by a particle Doppler dynamic analyzer (BSA P80, DANTEC Dynamics A/S), with a high response frequency of greater than 180 MHz and a high measurement accuracy of less than 0.05%. A three dimension displacement control system (Changcheng Institute of Metrology & Measurement, China) with a space error of ± 0.01 mm is installed for the PDA to obtain different radial locations accurately. A speed photoelectric sensor (Model WS-DNPW1, Zhongnuo, China) is used to calibrate motor speeds.

2.3. Fully-developed oscillating flow region

Three different axial velocities were measured by PDA for a certain radial position. It was defined a fully developed flow when each fluctuant value was within 5% of their average velocity. A few literatures studied the entrance length of fully-developed flow and proposed the criterion correlation [16,23,24], as Table 2.

In this study, the criterion of entrance length is based on the type of oscillatory flow. The ratio of length to diameter of PDA experiment pipe, i.e. L/D , is 15, which is beyond the range of the entrance length proposed by Zhao and Cheng [16].

3. Results and discussion

3.1. Analytical solutions of oscillating flow

The Bessel function describing field of oscillating flows was solved by 'Maple' soft for different Re_ω numbers. The results for a period of four different values of $Re_\omega = 0.083, 37, 900$ and 1800 were shown in Fig. 2. When the kinetic Reynolds number was 0.083, the maximum velocity was about 3.3 and the velocity profiles were similar to a steady flow (a parabola shape). For $Re_\omega = 900$ and $Re_\omega = 1800$, the maximum velocities were 1.9 and 1.7, occurring respectively, at locations of $r = 0.8$ – $0.9 R$, as shown in Fig. 2(c) and (d).

The maximum dimensionless oscillatory velocity $U_{\max}$ obtained by various investigators were listed in Table 3. The maximum velocities for various Re_ω were close, with 1.1–3.3. The discrepancies may be caused by the different forms of dimensionless velocities for different researchers.

3.2. Experimental results and discussion

To find suitable tracer seeds are a key problem for our PDA experiments, especially when the size and density of working fluid are both small. A suitable kind of particle should be traceable, scattering, clean, nontoxic, noncorrosive and abrasive. Four tracer particles used in PDA experiment are listed in Table 4.

The Stokes number (Stk) is a dimensionless number representing to the behavior of particles suspended in a fluid flow. It is defined as the ratio of the characteristic time of a particle to a characteristic time of the flow or of an obstacle [27].

$$Stk = \frac{\tau_p U_o}{D} = \frac{\rho_p d_p^2 / 18 \mu_g}{D / U_o} \quad (33)$$

For an acceptable tracing accuracy, the particle response time should be faster than the smallest time scale of the flow, which means that Stk number should be small. For $Stk \gg 1$, particles will detach from a flow especially where the flow decelerates abruptly; for $Stk \approx 1$, particles follow fluid streamlines closely; for $Stk \ll 1$, tracing accuracy errors are below 1% [28].

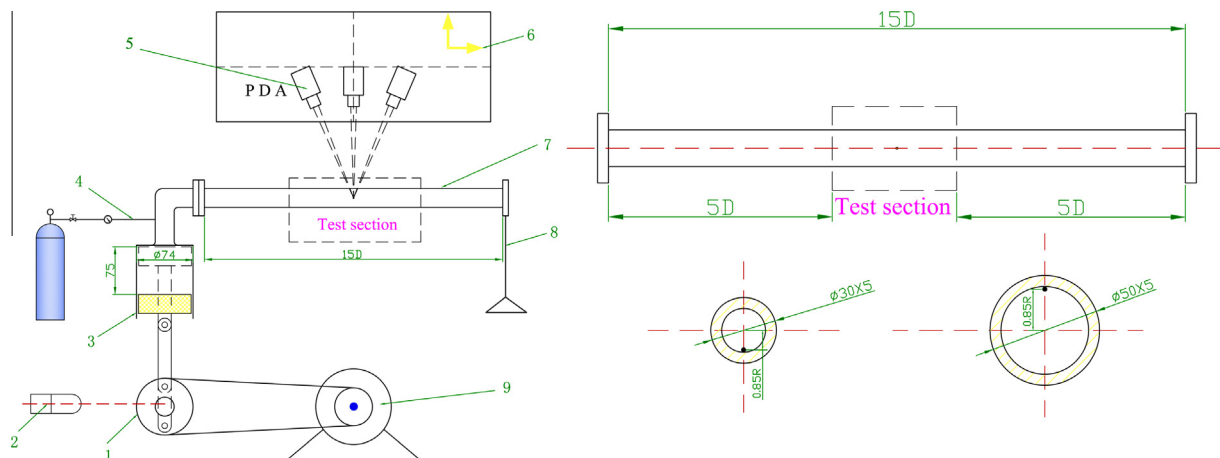


Fig. 1. PDA experiment rig. (a): System diagram: 1. Belt pulley. 2. Speed photoelectric sensor. 3. Cylinders and pistons. 4. Gas system. 5. PDA. 6. Three dimensional displacement control system. 7. Plexiglas pipe. 8. Support frame. 9. Asynchronous motor; (b): Configuration of test section.

Table 2
Dimensionless entrance length of steady and oscillatory flow.

Author	Flow types	The correlation for entrance length (L/D)
Chen et al. [23]	Steady flow	$\frac{0.63}{1+0.035Re_{max}} + 0.044Re_{max}$ (low Re numbers)
Zhao and Cheng [16]	Oscillatory flow	$0.081A + 0.23$
Sinclair [24]	Oscillatory flow	$[(0.725 Re_{max})^{2.4} + (0.06 Re_{max})^{2.4}]^{1/2.4}$

Considering, the aggregation feature of nano-fluids [29] and the PDA measurement range, SiO_2 of $<0.9 \mu m$ and $2.2 g/cm^3$ was chosen as tracer seeds for the PDA experiments. As to low- Re_{ω} experiments ($Re_{\omega} = 20$) [16], Bouvier et al. [30] provided a ratio of maximum velocities, i.e. 2.75, at shaft degrees of 300° and 0° . The ratio was very close to our analytical resolution, 2.8, indicating that our analytical resolutions were applicable for low-value Re_{ω} numbers.

In Fig. 3, the temporal velocity was evolved in the circle pipe during one cycle for different Re_{ω} numbers and positions, and error bars were also reported. The maximum dimensionless velocities for three different Re_{ω} were 1.9, 2.4 and 1.7, respectively. For each time, only one point can be measured by PDA, and experimental data need to be recorded by a time sequence. To reduce measurement errors, the average value of a shaft degree of 30° was chosen as an experimental result.

In Fig. 3(b), (c) ($Re_{\omega} = 900$), (d) and (e) ($Re_{\omega} = 1800$), they illustrate that the dimensionless velocity U near the wall ($r = 0.85 R$) was larger than that of core area ($r = 0$). The ratios of the maximum

velocity for $r = 0.85 R$ to $r = 0 R$ were 1.06–1.10, respectively, which was another evidence of “annular effect”. For a relatively small Re_{ω} (Fig. 3(b), $Re_{\omega} = 230$), experimental results were very close to analytical resolutions, except nearly-positive peak regions. With the increase of Re_{ω} , the deviation increases. The maximum deviations were about 23.5% and 24.0% for $r = 0$ and $r = 0.85 R$, respectively, at $Re_{\omega} = 900$, $D = 20$ mm. For $Re_{\omega} = 1800$ and $D = 40$ mm the maximum deviations were 24.8% and 21.2% for $r = 0 R$ and $r = 0.85 R$, respectively.

The deviation is probably caused by flowing factors. Firstly, the effect of inertial force is more important than that of viscous force, especially between two half periods. Secondly, it is the interaction forces among particles, as the size of seeds is bigger than gas molecular.

Besides, it needed to be mentioned that the maximum dimensionless velocity was 2.4 for $Re_{\omega} = 900$, while it was 1.9 for $Re_{\omega} = 230$, which contradicts the analytical resolution. This may be that they were in different flow region. A critical dimensionless parameter β_{cri} was used to describe the transition of oscillating flow [31],

$$\beta = A\sqrt{Re_{\omega}} = x_{max}\sqrt{\frac{\omega}{\nu}} \quad (34)$$

Zhao and Cheng [8] obtained a value of β_{cri} of 761. For our experiments, values of β are listed in Table 5, which indicates that Fig. 3(b) and (c) ($Re_{\omega} = 910$) are of no fully-developed of laminar flow. Therefore, the velocity model still needs to be modified in oscillating transition or turbulent region.

Finally, it was derived from the uncertain of PDA. Li, etc. [32] reported that PDA overall uncertainties of transient velocity and

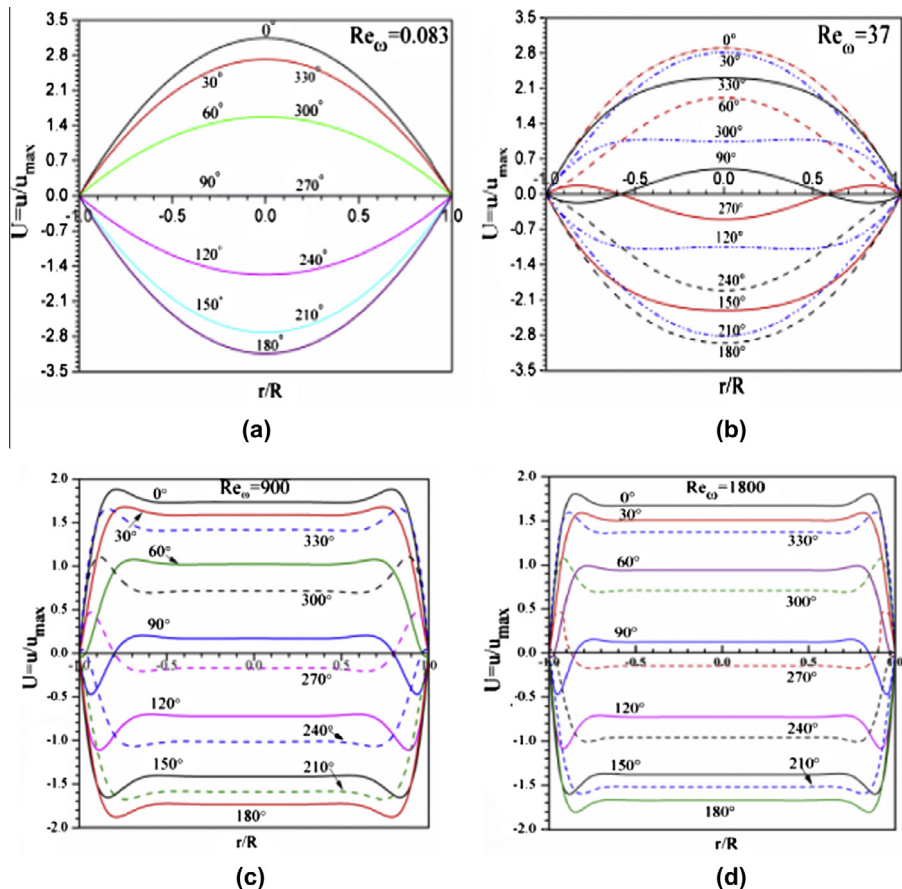


Fig. 2. Temporal axil velocity variation of a fully developed laminar oscillating flow at different radial locations. $Re_{\omega} = 0.083$, $D = 6$ mm; (b) $Re_{\omega} = 37$, $D = 10$ mm; (c) $Re_{\omega} = 900$, $D = 20$ mm; (d) $Re_{\omega} = 1800$, $D = 40$ mm.

Table 3Maximum dimensionless velocities of oscillating flows, $U_{\max}$.

Authors	Year	Re_{ω}	U	$U_{\max}$
Urich [10]	1956	4	$\frac{U}{\bar{U}}$	2.0
		1000	$\frac{x_{cr}/x_0}{u_{\max}}$	0.9
Zhao and Cheng [8]	1998	0.05	$\frac{u(r,t)}{u_{\max}}$	2.0
		200		1.35
		500		1.2
Sert and Beskok [25]	2003	1	$\frac{u(r,t)}{\bar{U}}$	2.3
		100		1.89
Liberto and Ciofalo [26]	2009	9	$\frac{u(r,t)}{u_0}$	1.17
		225		1.1
		0.083		3.3
Xiao etc.	2013	37	$\frac{u}{u_{\max}}$	2.8
		900		1.9
		1800		1.7

particle diameter were 1%, and 4%, respectively, when the particle size was within 0.5–1000 μm . Esteban etc. [33] indicated that the errors reached the maximum, $\sim 5\%$, for particles of 50 μm , and the errors was always less than 5% for other size particles. Based on the above researches, the uncertainty of our PDA measurement

Table 5Value of β of different situation.

D_i (mm)	A	Re_{ω}	β	β_{cri}
20	47	230	725	761
20	47	900	1050	761
40	6	1800	268	761

should be within 5% for particles of 900 nm. Besides, the compressibility of working fluid may also need to be considered. When a piston moved towards the top dead center, it bought a compression wave to travel from the piston face as a high pressure part of wave. The pressure wave steepened and affected the density of working fluid [34].

It was also found that nearly in all cases of Reynolds Numbers numerical simulation results did not show a good agreement with the experiments in position of 0 R. It was probably because that there were less tracer particles in the core region, i.e. the position of 0 R, than in near wall regions, then less experimental data and greater error were obtained. The laser of the PDA reached the core region through the near wall regions, which probably caused some optical errors, e.g. refraction. Furthermore, the maximum deviation occurred at the maximum velocity of the whole period, where the

Table 4

Tracer particle of PDA.

Tracer particle	Diameter (μm)	Density (g/cm^3)	$Stk = \frac{\rho_p d_p^2 / 18 \mu_s}{D / U_s}$	Technical comments
Glass bead	10–100	2.2–2.66	12.31	Too heavy, cannot float
Sulfur smog	≈ 50	2.55	8.85	The velocity of particle is about one or two order of magnitude smaller than theoretical value
SiO_2I	4–5	2.2	0.11	
SiO_2II	≈ 0.9	2.2	0.004	Close to the theoretical value

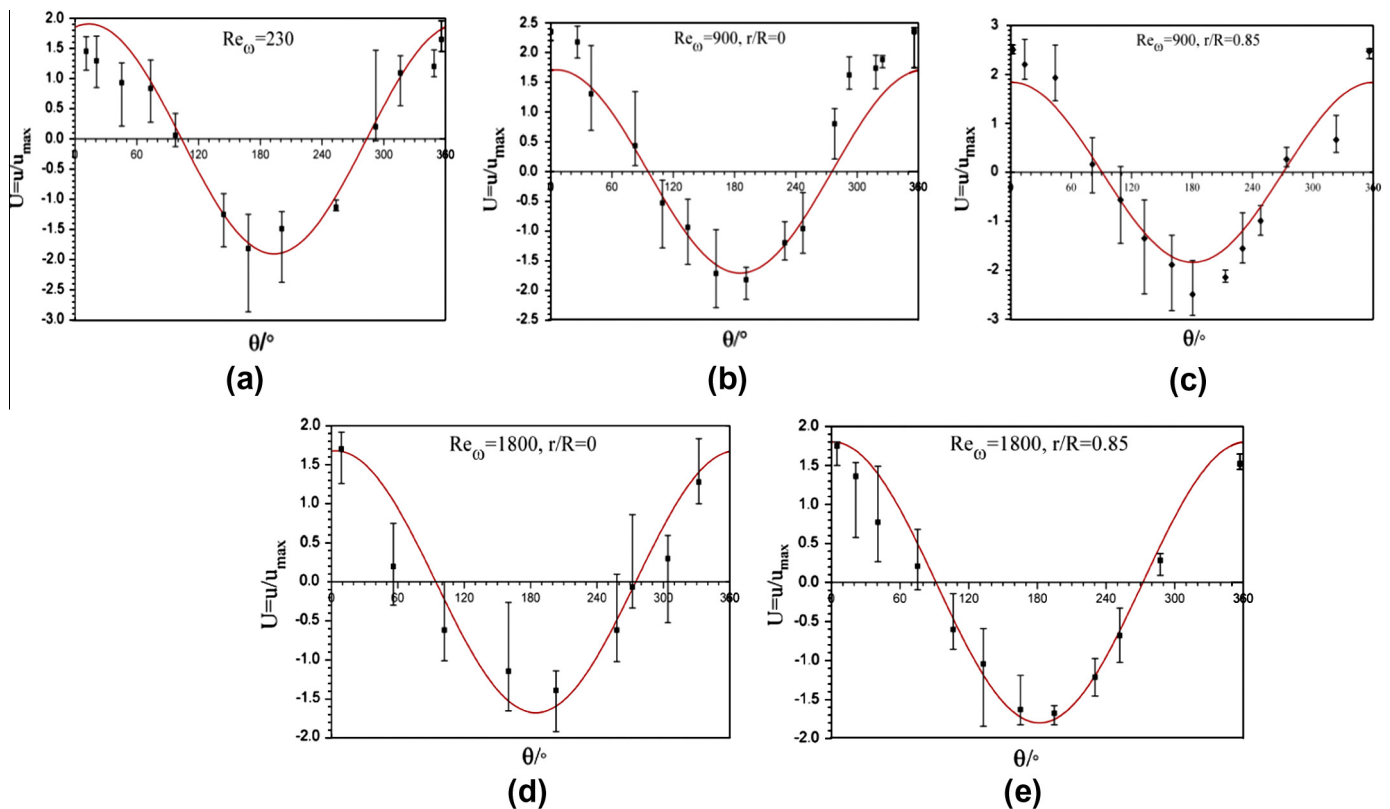


Fig. 3. Analytical profiles (lines) and PDA experimental profiles (symbols) of temporal dimensionless velocity U at various positions, center and 0.85 R of pipe for: (a) $Re_{\omega} = 230$, $D = 20$ mm; (b) $Re_{\omega} = 900$, $r/R = 0$, $D = 20$ mm; (c) $Re_{\omega} = 900$, $r/R = 0.85$, $D = 20$ mm; (d) $Re_{\omega} = 1800$, $r/R = 0$, $D = 40$ mm; (e) $Re_{\omega} = 1800$, $r/R = 0.85$, $D = 40$ mm.

Table 6
Comparison of experiments of oscillating flows.

Authors	Measuring equipment and their features	Re _{co} numbers and application fields	Purpose or results
Bouvier, et al. [30]	Photoelectric and LDA; smog, visualization; non-intrusive, tracer particles	5; –	Verify the analytical solution of low Re _{co} number laminar oscillatory flows
Zhao and Cheng [16]	HWA; Intrusive and real-time measurement; no tracer particles	0.1, 200, 500; 66.6, 302.2; Stirling engines and cry coolers	Study on transition from steady state to turbulence and frictional loss of oscillating flows
Friedman and Simon [36]	HWA; Intrusive and real-time measurement; no tracer particles	310.8; Stirling engines (SPRE)	Review and investigate transition mechanisms for a deep understanding boundary layer parameters of operating SPRE
Xiao et al.	PDA; different tracer particles (Silicon dioxide, Sulfur dioxide and glass bead), non-intrusive and real-time measurement	230, 900, 1800; Heaters and regenerators of Stirling engines, cryocoolers and gas springs	Verify the proposed complex velocity model and its analytical solutions of fully-developed laminar flows of moderate-value Re _{co} numbers.

inertial force of tracer particle brought a great deviation. Besides, the Re_{co}, i.e. frequency, of oscillating flows in our experiment was relative higher than other reports, which might lead to greater deviations. Fig. 3, indicated that the average deviation of all the PDA experimental profiles of temporal velocity was about 10%.

The experimental results may be affected by the instability of oscillating flow, and it is necessary to discuss these unstable factors. Firstly, it is the vibration of experimental rig. Based on the simple harmonic vibration equation [35], the acceleration of the experimental rig is given by,

$$a = (2\pi f)^2 A \times 10^{-5} \approx 0.004Af^2 \quad (35)$$

where A is the amplitude of displacement and f is the frequency of oscillation. The displacement of the experimental rig in z axis was measured by a displacement sensor and the maximum value is about 2.2 mm. When this value is substituted into the equation, the maximum value of acceleration is only 1.98 which is far less than the upper limit value 5. This indicates the effect of vibration of experimental rig can be neglected. Secondly, it is the effect of testing pipe elbow. Zhan and Cheng [8] reported that the velocity distribution became stable when x/D was greater than 10. Considering the ratio of length to diameter of PDA experiment pipe is $x/D = 15$, larger than above the value 10, the effect of elbow is insignificant. Finally, it is the rotation of tracer particles in oscillating flow field. For there is no paper, as we know, reported the study on the influence of tracer particle rotation. Based on the experimental results and our on-site observation, the influence of rotation is considered to be slight and negligible.

As above mentioned, there are three main new points in this paper. (1) A new method of PDA was proposed to detect oscillating flows experimentally. Compared with other methods, e.g. HWA, photoelectric, LDA and/or PIV, PDA cannot only obtain temporal velocity distributions of oscillating flow fields, but provide concentration and size distributions of tracer particles. Furthermore, three kinds of particles of different sizes and densities were tested and analyzed, and SiO₂II was suggested as the tracer particles. (2) The experimental range of kinetic Reynolds numbers (Re_{co}) was extended, 0–1800 in our experiment while 0–500 in other reports, as listed in Table 6. (3) It is also noticed that the deviation between the analytical resolution and experimental results became great in the oscillating transition or turbulent region, and further work is suggested in the future.

4. Conclusion

A complex approach was proposed based on the hypothesis of fully-developed laminar oscillating flow to solve control equations and to gain velocity distributions of oscillating flow under various conditions. The velocity profiles for Re_{co} = 0.083, 37, 900 and 1800 were described and their maximum dimensionless velocity were 3.3, 2.8, 1.9 and 1.7, respectively.

For analytical solutions, an oscillating flow of low-value Re_{co} (0.085) was similar to a steady flow. An “annual effect” occurred at $r = 0.8$ – 0.9 R of a relatively high Re_{co}, instead of the core section. The cross-sectional maximum dimensionless velocity decreased and the smooth core area of velocity expanded as Re_{co} increased, leading the Stokes layer became thinner.

A new method of PDA was proposed to study oscillating flows experimentally and SiO₂ of 0.9 μm diameter was suggested as tracer particles. Meanwhile, the experimental range of kinetic Reynolds numbers (Re_{co}) was extended in 0–1800. The average deviation between analytical solutions and experimental results was about 10%, which indicated that complex velocity model was suitable to describe a fully-developed laminar oscillating flow. For a moderate-value Re_{co}, the ratios of maximum dimensionless velocity for $r = 0.85$ R to $r = 0$ R were 1.06–1.10, which indicated there was “annual effect”.

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